

QUESTIONS AND PROBLEMS.

1) About the Y_k -equivalence relation.

- Characterization of the Y_k -equivalence in terms of well-understood invariants :

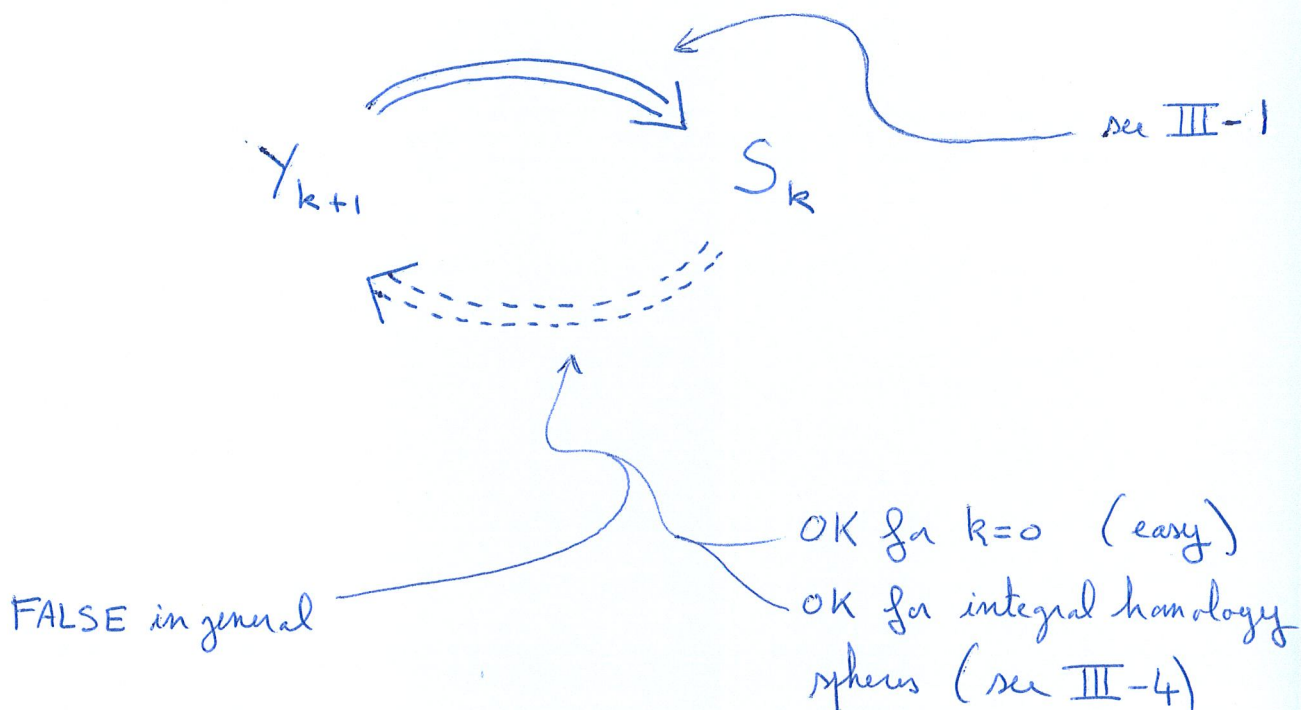
OK for $k=1, 2$ (see II-5)

Question. for $k \geq 3$?

- Y_k -equivalence vs finite type invariants :

Def.

Two compact oriented 3-manifolds are S_k -equivalent if they are not \neq by FTI of degree $\leq k$.



Example. (the simplest I know...)

$$M = \#^4 S^2 \times S^1$$

$$M' = S^1 \times S^1 \times S^1 \# S^2 \times S^1$$

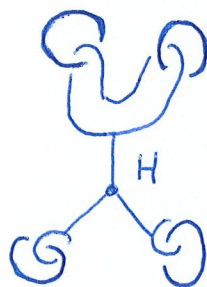
$M \not\cong_{Y_2} M'$ since their cohomology rings are not v.

$M \cong_{S_1} M'$, which is seen as follows:

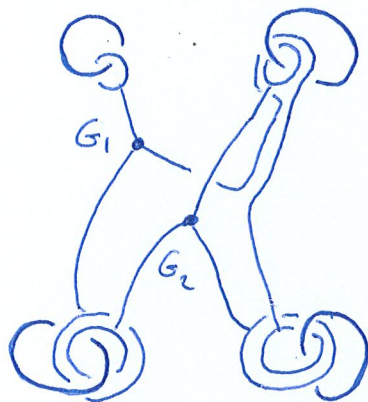
$M = S^3 \circ 0000 \leftarrow$ trivial link with 4 components

$M' = S^3 \circ 0 \leftarrow$ bocomean rings \cup a trivial knot

Here is a Y -class $H \subset M$:



Here are two Y -classes $G_1, G_2 \subset M$:



$$[M; G_1 \cup G_2] = M - M_{G_1} - M_{G_2} + M_{G_1 \cup G_2}$$

$$M_{G_1} \cong M' \cong M_{G_2}$$

$$M_{G,UG_2} \stackrel{\cong}{=} M_H \stackrel{\cong}{=} M'$$

\uparrow by Move II, see II-2 \uparrow by a Kirby move

Questions.

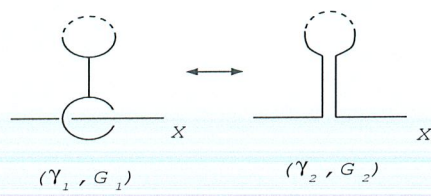
- How is Y_{k+1} far from S_k ?
- $Y_\infty = S_\infty$?
- $Y_{k+1} = S_k$ for homology cobordisms ?
 (conjectured by Goussarov and Habiro ; OK for $k = \infty$)

2) About the universal invariant.

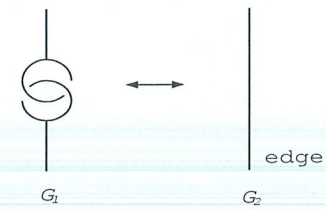
For b_1	Z^{LMO} is equivalent to	
$= 0$	all rational-valued FTI	(Habiro, Garoufalidis)
$= 1$	Conway polynomial	(Garoufalidis - Habegger, Lieberman)
$= 2, 3$	Casson-Walker-Lescop inv.	(Beliakova-Habegger, Habegger)
≥ 4	0	

Problem. Improve the LMO's construction (or, equivalently, the BGRT's one) of a universal invariant from the Kontsevich integral, which would not weaken so much when $b_1 \nearrow$.

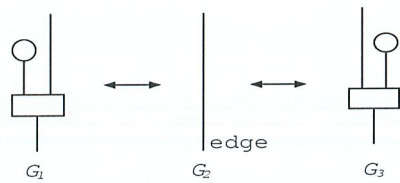
MORE QUESTIONS IN
OHTSUKI'S LIST OF PROBLEMS...



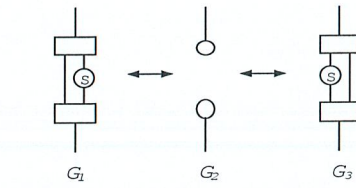
move 1



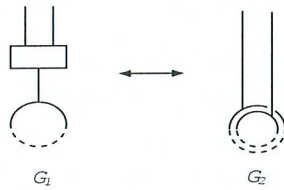
move 2



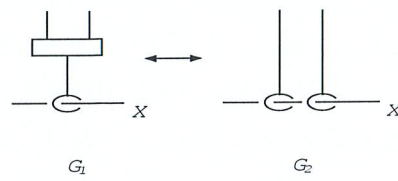
move 3



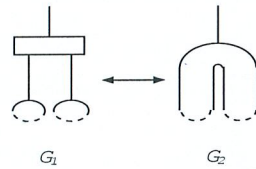
move 4



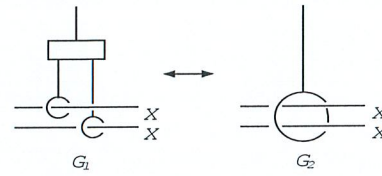
move 5



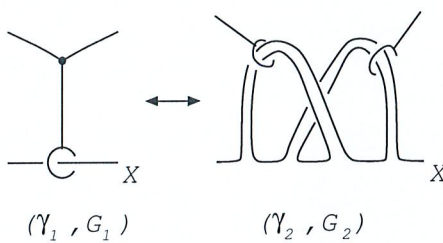
move 6



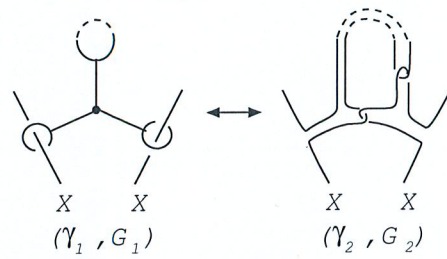
move 7



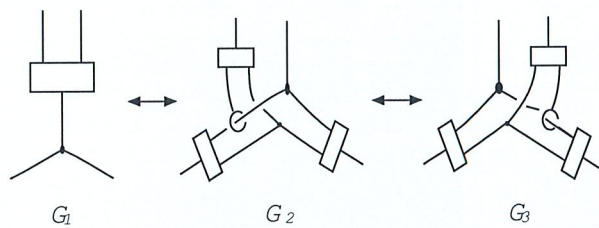
move 8



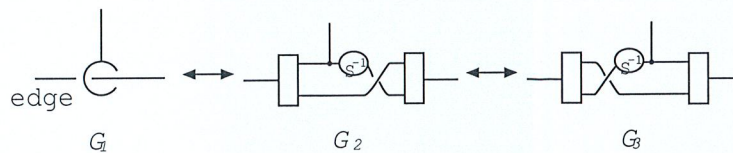
move 9



move 10



move 11



move 12

Habiro's Twelve Moves. (Here, X represents a parallel family of strings of a link, edges or leaves of claspers.)

References

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